# The Application of the Geddes Fractionation Index to Crude Distillation Units

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The degree of separation effected by a fractionator can be measured by a fractionation index n. The index is the equivalent number of theoretical plates, operating at total reflux, which would effect the same component separation as the fractionator.

The value of the fractionation index is readily calculated from the Fenske-Underwood equation when component analyses of the feed and products are available.

In the case of crude oils, the number of components is so great that resolution in the laboratory is not practicable, and the feed and products are customarily analyzed by high efficiency laboratory fractionators which give their true boiling point characteristics. In this form, the data have been broken down into short fractions which have then been treated as pseudo components in subsequent column calculations.

This paper presents a method for treating the product TBP curves as algebraic functions without the necessity for breaking the feed TBP data into arbitrary short fractions. The fractionation index can be derived directly from product TBP data, or the TBP data can be calculated when the fractionation index is known.

Data from performance tests on three commercial units have been analyzed. The TBP distillation analyses of the crude oils and the products enabled the fractionation indices pertaining to each section of the fractionators to be evaluated.

Calculation methods for the fractionation of multicomponent hydrocarbon systems have been the subject of numerous papers, and the design of columns to separate complex mixtures of specific components can now be carried out reasonably easily using computer programs.

The fractionation requirements for refinery streams, particularly products from crude oil distillation units, are nevertheless widely defined in terms of ASTM distillation data and very empirical methods are still used in defining the fractionating ability of operating refinery units (1). Relatively few papers have compared the properties of refinery streams measured by conventional methods with the calculated performance of the fractionation equipment producing the streams.

The basic requirement for defining the quality of a crude distillation product stream is a knowledge of its true boiling point (TBP) characteristics. Specific gravity, octane number, and sulfur content, etc., can then be calculated, provided that sufficiently detailed information is available on the crude oil considered.

Cecchetti and co-workers (2) showed good agreement between calculated and test run TBP data for a crude distillation unit. In this case the crude oil feed was broken down into thirty-four pseudo components, but this number of components is rather small if detailed TBP curves are required for five or six overhead and sidestream products.

This paper presents a means of using a fractionation index to predict TBP data for the products from a crude oil distillation unit. The algebraic treatment was facilitated by making some simplifying assumptions, but experimental results show that these simplifications are justifiable, at least in the case of the Middle East crude oils processed in the experimental work.

## BASIS OF METHOD

TBP data for the products from a crude oil distillation unit can be superimposed, or placed, on the TBP data for the crude oil feedstock as shown in Figure 3. From such a diagram the TBP characteristics, and hence most of the properties required for product specification, can be determined.

Product properties are controlled by the initial and final cut point of the product and are modified by the degree of fractionation provided for the separations at the cut points. No simple method has previously been available for defining the shape of the product TBP curve in terms of the column fractionating ability. For refinery planning, the properties of cuts have usually been determined from crude oil TBP data in conjunction with the cut points it is proposed to use, and some arbitrary allowance made for normal refinery fractionation.

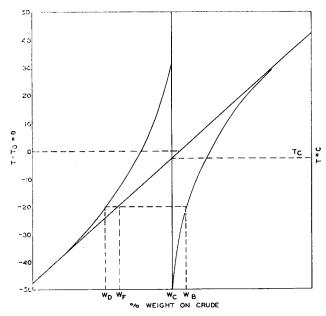


Fig. 1. Typical separation of feedstock TBP into two products at cut point  $T_C$ .

TABLE 1. TBP DISTILLATION OF PRODUCTS FROM EQUILIBRIUM FLASH OF TOPPED KUWAIT CRUDE

Flash condit	rions Temperature, °C. Pressure, lb./sq. in. abs.	280 14.7	
Cut point	% wt. on topped crude Temperature. °C.	30.30 269	

Boiling temper- ature, °C.	θ	Position on topped crude feed WF % wt.	topped	ion on d crude % wt. Liquid, WB	$\frac{Wc - W_D}{W_F - W_C}$	, <b>n</b>
200	69	19.25	17.73	31.82	-1.13766	1.10
210	-59	20.80	18.94	32.17	-1.19666	1.10
220	-49	22.37	20.09	32.58	-1.28785	1.09
230	39	23,98	21.21	33.07	-1.43747	1.09
240	-29	25.63	22.35	33.59	-1.70365	1.08
250	-19	27.27	23.33	34.24	-2.30237	1.06
260	9	28.84	24.24	34.90	-4.14824	1.08
<b>26</b> 9	0	30.30	25.01	35.59		1.08
270	+1	30.45	25.09	35.66	+35.14224	1.08
280	+11	31.95	<b>2</b> 5.83	36.42	+2.70632	1.02
290	+21	33.50	26.48	37.32	+1.19234	1.03
300	+31	35.07	27.12	38.25	+0.66672	1.05
310	+41	36.64	27.65	39.29	+0.41821	1.06
320	+51	38.13	28.10	40.33	+0.28050	1.07
330	+61	39.73	28.56	41.47	+0.18481	1.09
340	+71	41.28	28.94	42.65	+0.12416	1.12

A typical separation taking place at cut point  $T_C$  °C. and  $W_C$  wt. % on crude is shown in Figure 1. The shapes of the product TBP curves are controlled by the distribution of components having normal boiling points within about  $\pm 75\,^{\circ}$ C. of the component corresponding with the cut point temperature. If the equations controlling the distribution of these components can be defined, then equations representing the product TBP curves may be written.

The crude oil feedstock can be regarded as being composed of an infinite number of components any one of which, denoted by suffix i, can be identified by its boiling point  $T_i$ . One component  $T_o$  will be equally distributed between the two products, and it is convenient to define the relative volatility of this component as unity. Any other distributed component can be identified by the difference  $\theta_i$  between its boiling point and that of the equally distributed reference component.

Under total reflux conditions, the distribution of any component i having volatility  $\alpha_i$  relative to the equally distributed component is given by

$$\alpha_i^n = \frac{\Delta X_D}{\Delta X_B} = \frac{\Delta W_D}{\Delta W_B} \tag{1}$$

When separation is effected at finite reflux ratios there is a corresponding total reflux operation which defines the component distribution for many practical purposes. Geddes (3) suggested that the exponent of  $\alpha_i$  for this equivalent total reflux operation could be used as an index of fractionating power. He showed that when a multicomponent mixture is distilled a linear plot of slope n is ob-

tained for  $\log \alpha_i$  against  $\log \left(\frac{X_D}{X_B}\right)_i$ ,  $\alpha$  being unity for

the equally distributed component.

The distribution of any component i can therefore be expressed for the present purpose in terms of the fractionation index n proposed by Geddes and its relative volatility  $\alpha_i$ . The use of the temperature difference  $\theta_i$  to define the normal boiling temperature of a distributed

component permits the application of an approximate relationship which need only be true over a limited boiling point range

A suitable relation is provided (4, 5) by combining Trouton's rule with the Clausius-Clapeyron equation.

$$\ln \frac{P_1}{P_0} = \frac{-10.59}{273 + T} \left( T_i - T_o \right) \tag{2}$$

For separations effected in crude oil distillation units operating at 0 to 50 lb./sq. in. gauge,  $\alpha_i$  may be expressed as  $\frac{P_i}{P_o}$  and T is approximately the same as the cut point temperature  $T_c$ , so that for distributed components

$$\ln \alpha_i = \frac{-10.59}{273 + T_c} \theta_i = -A\theta_i \tag{3}$$

where A is a constant which characterizes the distribution of components in the vicinity of the cut point temperature T.

Equations (1) and (3) can be combined to give the required distribution of any component *i* whose normal

TABLE 2. UNIT A. TBP DATA FOR PRODUCTS

	Gas- oline*	Naph- tha*	Kero- sene*, †	gas oil*,†	Resi- due†	
Temp.,	Posi- tion on crude,		Posi- tion on crude,	Posi- tion on crude,	Posi- tion on crude,	Crude oil*, † Iranian
°C.	% wt.	% wt.	% wt.	% wt.	% wt.	light
100	9.24	17.56				9.6
110	10.63	17.59				10.6
120	11.88	17.65	-			12.2
130	13.52	17.76				13.4
140	15.22	18.03	25.06			15.7
150	16.25	18.63	25.09			17.5
160	17.11	19.67	25.15			19.3
170	17.41	21.31	25.29			21.1
180	17.54	22.89	25.54			22.9
190	1.101	23.91	26.06			24.2
200		24.55	27.00			26.5
210		24.87	28.28	37.82		28.6
220		25.00	30.01	37.84		30.2
230		25.05	31.73	37.91		32.0
240			33.42	38.05		33.8
250			35.10	38.30		35.6
260			36.38	38.76	53.28	37.4
270			<b>37.2</b> 1	39.78	53.33	39.2
280			37.56	41.30	53.37	41.0
290			37.74	42.84	53.42	42.8
300				44.39	53.56	44.6
310				45.97	53.75	46.4
320				47.51	54.07	48.3
330				49.07	54.54	50.0
340				50.28	55.20	51.8
350				51.16	55.99	53.6
360				51.81	56.92	<b>55.4</b>
370				52.32	58.05	57.2
380				52.69	<b>59.26</b>	58.9
390				52.92	60.57	60.5
400				53.09	62.02	62.2
410				53.20	63.65	63.8
420				53.26	65.33	65.4
430					67.11	66.9
	17.56	25.06	37.82	53.28	100.00	

Laboratory Fractionators

e Forty theoretical plates; 40:1 reflux ratio; 60:1 vol charge/hold-up ratio.
† Fifteen theoretical plates; 20:1 reflux ratio; 50:1 vol. charge/hold-up ratio.

				TAB	LE 3. UNIT	A				
Cut	$W_C$ , % wt.	Boiling		$W_D$	$W_B$	$\frac{Wc-Wp}{}$			$W_D$	$W_B$
point, °C.	on crude	temp., °C.	$W_{F}$	Experi	mental	$W_F - W_B$	$\boldsymbol{n}$	Mean n	Calc	ulated
		110	10.66	10.63	17.59	-1.00435	3.94 )		10.64	17.58
		120	11.97	11.88	17.65	-1.01610	3.87		11.90	17.63
		130	13.72	13.52	17.76	-1.05208	4.07	4.09	13.52	17.76
		140	15.69	15.22	18.03	-1.25134	4.06	4.00	15.23	18.02
151.1	17.56	150	17.35	16.25	18.66	-6.23807	4.53		16.12	18.79
		151.1	17.56	16.34	18.78	0.20001	4.47		16.34	18.78
		160	19.31	17.11	19.76	0.25714	4.98 €		17.11	19.76
		170	21.39	17.41	21.54	0.03916	4.96	4.97	17.41	21.54
		160	19.31	19.22	25.15	-1.01565	4.29		19.22	25.15
		170	21.39	21.16	25.29	-1.06267	4.20		21.17	25.28
		180	23.35	22.87	25.54	-1.28070	4.24	4.30	22.88	25.53
		190	24.91	23.91	26.06	-7.66667	4.47		23.87	26.10
190.9	25.06	190.9	25.06	23.97	26.15	1.00001	4.41		23.97	26.15
		200	26.49	24.55	27.00	0.35664	4.51 )		24.58	26.13 26.97
		210	28.09	24.87	28.28	0.06271	4.62	4.66	24.87	28.28
		220	29.97	25.00	30.03	0.01222	4.85	4.00	24.99	30.04
		230	31.81	31.72	37.91	-1.01498	4.74		31.74	37.89
		240	33.65	33.42	38.05	-1.05516	5.69		33.46	38.01
		250	35.58	35.10	38.30	-1.21429	4.86	5.00	35.13	38.27
		260	37.32	36.38	38.76	-2.88000	5.69		36.22	38.92
262.6	37.82	262.6	37.82	36.60	39.04	-2.00000	5.45		36.60	39.04
		270	39.22	37.21	39.83	0.43571	5.70 )		37.18	39.86
		280	41.13	37.56	41.39	0.07855	5.40 }	5.56	37.58	41.73
		290	42.90	37.74	42.98	0.01575	5.59	0.00	37.74	42.98
		280	41.13	41.04	53.37	-1.00741	3.18		41.03	53.38
		<b>2</b> 90	42.90	42.76	53.42	-1.01349	3.25		42.73	53.45
		300	44.67	44.39	53.56	-1.03252	3.12		44.38	53.57
		310	46.44	45.97	53.75	-1.06871	3.12	3.08	45.95	53.77
		320	48.30	47.51	54.07	-1.15863	3.08	0.00	47.51	54.07
		330	50.33	49.07	54.54	-1.42712	2.99		49.13	54.48
		340	52.20	50.28	55.20	-2.77778	2.80		50.49	54.99
346.5	53.28	346.5	53.28	50.85	55.71	-2.77710	2.79		50.85	55.71
	<b>33.2</b> 3	350	53.87	51.16	55.99	3.59322	2.82		51.23	55.92
		360	55.45	51.81	56.92	0.67742	2.72		51.95	56.78
		370	57.09	52.32	58.05	0.25197	2.79		62.40	57.97
		380	58.67	62.59	59.26	0.10946	$\frac{2.79}{2.79}$	2.91	62.72	59.23
		390	60.21	52.92	60.57	0.05195	2.88	2.01	52.93	60.56
		400	61.83	53.09	62.02	0.02222	3.02		53.06	62.05
		410	63.57	53.20	63.65	0.00777	3.29		53.14	63.71
		710	00.01	00.20	00.00	0.00111	0.20 )		50.13	00.1 £

boiling point differs by  $\theta$ °C. from that of the equally distributed component.

$$\frac{\Delta W_D}{\Delta W_R} = e^{-nA\theta} = e^{-\psi} \tag{4}$$

The equations for the product TBP curves can be written in differential form in terms of the slope C of the crude oil TBP curve.

For overheads product:

$$\frac{d\theta}{dW_D} = C(1 + \alpha^{-n}) \tag{5}$$

For bottoms product:

$$\frac{d\theta}{dW_{\rm B}} = C(1 + \alpha^n) \tag{6}$$

Before these equations can be applied it is necessary to identify the normal boiling point of the equally distributed reference component.

From Equations (3) and (5)

$$-ACdW_{D} = \frac{\alpha^{n-1}}{1+\alpha^{n}}d\alpha \tag{7}$$

Integrating between the limits

$$\alpha = 0 \qquad W_D = W_C$$

$$\alpha = \alpha \qquad W_D = W_D$$

$$AC(W_C - W_D) = 1/n \ln (1 + \alpha^n)$$
 (8)

The equation for the bottoms curve corresponding to Equation (7) is

$$-ACdW_B = \frac{\alpha^{-(n-1)}}{1 + \alpha^{-n}} d\alpha$$
 (9)

Integrating between the limits

$$\alpha = \infty \qquad W_B = W_C$$

$$\alpha = \alpha \qquad W_B = W_B$$

$$AC(W_B - W_C) = 1/n \ln (1 + \alpha^n) - \ln \alpha \qquad (10)$$

Comparison of Equations (8) and (10) shows that when  $\alpha = 1$ , then

$$W_C - W_D = W_B - W_C$$

But from the mass balance equation for all  $\alpha$ 

$$W_C - W_D = W_B - W_F$$

Hence, when  $\alpha=1$ ,  $W_F=W_C$ . Thus the equally distributed reference component is identified as component  $W_C$ .  $T_C$ .

Equations (5) and (6) can now be integrated and

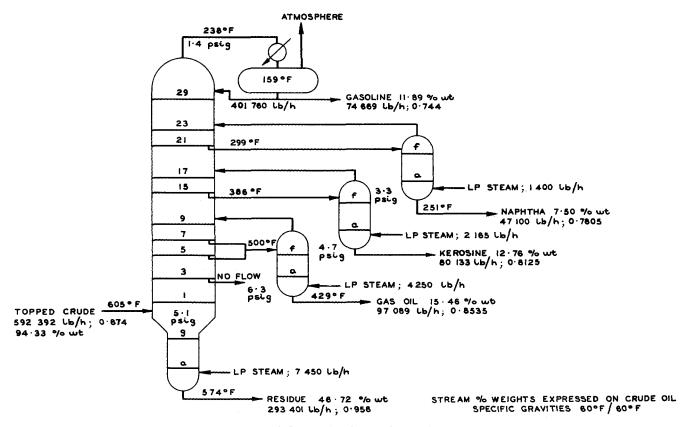


Fig. 2. Process flow diagram for unit A.

located on the crude TBP curve, Equation (11), to give Equations (12) and (13) for the distillate and bottoms products, respectively.

$$W_F - W_C = \frac{\theta_i}{C} \tag{11}$$

$$\frac{W_C - W_D}{W_F - W_C} = \frac{\ln (1 + e^{-\psi})}{\psi} = f(\psi)$$
 (12)

and

$$\frac{W_B - W_C}{W_F - W_C} = 1 + \frac{\ln(1 + e^{-\psi})}{\psi} = 1 + f(\psi) \quad (13)$$

The value of  $\psi$  can be determined for any distributed component identified by the point  $W_F$  on the crude TBP curve from the corresponding value of either  $W_D$  or  $W_B$ . Both values will normally be available from the distillate and bottoms product TBP curves, but errors in the location of the cut point  $W_C$  usually lead to differences between the values of  $\psi$  calculated from the vapour and liquid curves, for the same product distribution. The material balance equation

$$W_B + W_D = W_F + W_C$$

enables a value of  $W_F$  to be calculated which is consistent with the experimental values of both  $W_D$  and  $W_B$  so that an average value of  $\psi$  can be obtained from either the vapour or liquid TBP data.

At the cut point temperature, the degree of fractionation for a crude of slope C can be defined by the length of the intercept  $W_B - W_D$ . From Equations (8) and (10), when  $\alpha = 1$ 

$$W_C - W_D = W_B - W_C = \ln 2/nAC$$
 (14)

The temperature difference  $C(W_C - W_D)$  or  $C(W_B - W_C)$  was proposed by Meyer (6) as a measure of frac-

tionation efficiency (overlap coefficient  $\phi$ ) and defined by him as

$$\phi = \frac{1}{2} C(W_B - W_D)$$
 (15)

The overlap coefficient is easily obtained from product TBP data and its relationship to the index n can be derived from Equation (14)

$$\phi = \ln 2/nA \tag{16}$$

#### APPLICATION OF THE EQUATIONS

# Equilibrium Flash Data

The ability to predict the vapor-liquid phase behavior of crude oil components using the relationship between boiling point difference and relative volatility

$$\ln \alpha = \frac{-10.59}{273 + T} \theta_i \tag{3}$$

is capable of simplifying many refinery calculations. A direct experimental check was therefore made by carrying out a laboratory equilibrium flash separation of topped Kuwait crude and determining TBP data for the vapor and liquid products. The results are summarized in Table I.

From experimental values of  $W_D$ ,  $W_C$ , and  $W_B$ , values of the function  $\frac{W_C - W_D}{W_F - W_C}$  were calculated, so that  $nA\theta$  could be evaluated. The calculated values of n are reasonably constant and approximate closely to unity, thus confirming the applicability of Equation (3) and the value of the constant which it contains.

The derivation of a single constant from flash data in this way provides a convenient means of characterizing the vapor-liquid phase behavior of complex hydrocarbon mixtures.

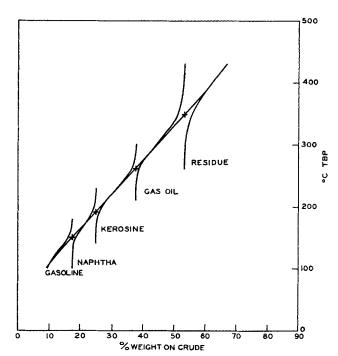


Fig. 3. TBP curves of products Ex unit A superimposed on column feedstock.

#### Crude Oil Distillation Units

Test run product TBP data for three crude distillation units were used to calculate fractionation indices for the separations taking place in the units. The data are exemplified by Tables 2 and 3, which give the experimental TBP data and derivation of fractionation indices for unit A° (Figure 2). The positions of the product TBP curves on the crude oil TBP curve are shown in Figure 3.

When he proposed the use of n as an index of fractiona-

When he proposed the use of n as an index of fractionation, Geddes showed that in practical column operation the rectifying and stripping functions of the column are not usually in balance. The distribution of components lighter than the reference component is controlled by the stripping power of the column and that of heavier components by its rectifying power. In general he obtained two intersecting straight lines for the  $\log \left(\frac{X_D}{X_B}\right)_i$  vs.  $\log \alpha_i$  plot

TABLE 4. FRACTION INDICES

		Fractionation Cut index n		
Unit	Separation	point, °C.	$\theta$ Negative	$\theta$ Positive
A Crude oil Iranian light	Gasoline/naphtha Naphtha/kerosene Kerosene/gas oil Gas oil/residue	151 191 263 347	4.1 4.3 5.0 3.1	5.0 4.7 5.6 2.9
B Crude oil Iranian light	Gasoline/naphtha Naphtha/kerosene Kerosene/lt. gas oil Lt. gas oil/hy. gas oil Hy. gas oil/residue	147 170 222 310 345	4.5 4.7 4.4 2.6 1.9	6.0 7.7 6.2 3.5 2.6
C Crude oil Kuwait	Gasoline/naphtha Naphtha/kerosene Kerosene/lt. gas oil Lt. gas oil/hy. gas oil Hy. gas oil/residue	138 193 217 309 339	4.8 4.25 3.8 2.2 1.85	7.0 6.9 7.2 2.7 1.75

with two values for the fractionation index n, corresponding with the effectiveness of the column stripping and rectifying functions.

The design of a crude distillation unit is unlikely to result in an exact balance between its stripping and rectifying power for any of the separations effected. The results obtained in fact indicate this expected difference in effectiveness of the column functions, and in Table 3 mean values of n are calculated separately for components lighter and heavier than the equally distributed reference component. Calculated values of the fractionation index for the three crude units considered are summarized in Table 4.

In general, n values for constituents boiling below the cut point temperatures are lower than those boiling above the cut point temperatures. This reflects the relatively poor fractionating ability of open steam strippers.

The mean values of n relating to component distributions above and below the cut points were used to reconstruct the original product TBP data as shown in Table 3. Agreement is generally good, showing that if appropriate values of the fractionation index are chosen product TBP data can be predicted with sufficient accuracy for all refinery purposes.

When a value for the fractionation index is available, TBP data for distillate products can be calculated from the feed TBP data as illustrated in the following example for naphtha-kerosene separation (unit B).

W <sub>F</sub> % wt.				$\frac{W_C - W_D}{W_F - W_C} \text{ or } $ $\frac{W_B - W_C}{W_B - W_C} - 1$	Calcu	ılated	Experi	mental
on crude	Temp., °C.	θ	nA θ	$W_F - W_C$	$W_D$	$W_B$	$W_D$	$W_B$
13,60	130	38.9	4.380	1.001	13.59	20.29	(13.56)	20.28
15.40	140	28.9	-3.254	1.012	15.34	20.34	(15.52)	20.34
17.10	150	-18.9	-2.128	-1.053	16.93	20.45	(17.15)	20.44
18.90	160	8.9	-1.002	-1.312	18. <b>47</b>	20.71	(18.35)	20.61
20.28	168.9					_		
20.50	170	+1.1	+0.203	+2.942	19 <b>.6</b> 3	21.15	19.49	(21.10)
22.10	180	+11.1	+2.048	+0.059	20.17	22.21	20.19	(22.50)
23.80	190	+21.1	+3.893	+0.005	20.26	23.82	20.26	(23.71)

Corresponding data from units B and C have been deposited as document 8676 with the American Documentation Institute, Photo-duplication Service, Library of Congress, Washington 25, D. C., and may be obtained for \$1.25 for photoprints or 35-mm. microfilm.

Crude TBP data	% wt.	Temperature, °C.	
	13.60	130	
	15.40	140	
	17.10	150	
	18.90	160	
Cut point	20.28	168.9	
•	20.50	170	
	22.10	180	
	23.80	190	
n Value Strippi	ng 4	1.7	
Rectify	ing 7	7.7	
		6 .7	

The calculation gives the results of the separation of crude oil into two products at a cut point of 168.9°C. and in this particular case some components were distributed between three products, gasoline, naphtha and kerosene. The experimental data in brackets represent combined data for gasoline plus naphtha, and kerosene plus gas oil, so that only the separation at 168.9°C. is considered.

The consistency of the results obtained for the particular crude oils and distillation units discussed in this paper shows that the analytical treatment of TBP data may be used to designate the fractionation efficiency of refinery equipment and to predict the qualities of refinery streams.

The method of analysis was developed specifically for use with Middle East crude oils having the required linear TBP characteristics. Its application to crude oils which, although linear with respect to the TBP/wt. % relation may have very different hydrocarbon component characteristics from Middle East crudes, would require a laboratory check by equilibruim flash experiments. It is possible that the constants used in Equation (3) relating relative volatility to boiling point difference may not be the same for all types of crude oil.

# **ACKNOWLEDGMENT**

The authors wish to thank the chairman and directors of the British Petroleum Co., Ltd., for permission to publish this

#### NOTATION

constant in Equation (3), equal to 10.59/273+T,

= slope of TBP curve for crude oil, °C./% wt.

= number of theoretical plates at total reflux = component vapor pressure, lb./sq. in. abs.

T = boiling point, °C.

X = molar flow rate, mole/hr.

= wt. % based on feed

= relative volatility

 $= T - T_o$ , °C.

= overlap coefficient,  $\frac{1}{2}$   $C(W_B - W_D)$ , °C.

= argument of function in Equation (13), equal to

# Subscripts

= bottoms product В

C= cut point

D= distillate

F = feed

= component i

= reference component

## LITERATURE CITED

 Packie, J. W., Trans. A.I.Ch.E., 37, 51-78 (1941).
 Cecchetti, Ralph, R. H. Johnston, J. L. Niedzwiecki, and C. D. Holland, Hydrocarbon Processing, 42, (9), 159-164 (September, 1963).

3. Geddes, R. L., A.I.Ch.E. J., 4, (4), 389-392 (1958).

Butler, R. M., and I. S. Pasternak, Can. J. Chem. Eng., 47-53 (April, 1964).

5. Butler, R. M., G. M. Cooke, G. G. Lukk, and B. G. Jameson, Ind. Eng. Chem., 48, (4), 808-811 (April, 1956).
6. Meyer, P., J. Inst. Petrol. Technol., 19, 819-834 (1933).

Manuscript received July 28, 1965; revision received November 8, 1965; paper accepted November 17, 1965.

# Turbulent Flow in Annular Pipes

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An extension to annular conduits of the well-known method of calculating the resistance coefficient for turbulent flow in circular pipes is presented. The method is new in that it does not borrow any information from laminar flow theory; it gives, therefore, a location for the maximum velocity based only on turbulent flow characteristics. Expressions and a few essential graphs are given for the average as well as inner- and outer-wall resistance coefficients for both hydraulically smooth and rough regimes and also for the transitional regime. A comparison with available experimental information for fully developed turbulent flow is presented.

Ever since the theoretical approaches of Prandtl and Kármán led to the successful synthesis of the perplexing experimental information on uniform flow in circular pipes then available, the logarithmic velocity distribution has been regarded by many as a very general law applicable to a rather wide range of situations. That this could be

done only as an approximation was obvious from the assumptions and calculations that led to the adoption of the logarithmic velocity distribution for circular pipes (1). Applications have been made to different forms of the cross section (2) for which there is no assurance that the secondary flow is negligible. More favorable cases are pro-